# Electrical and Electronic Theory and Technology 10-6-19

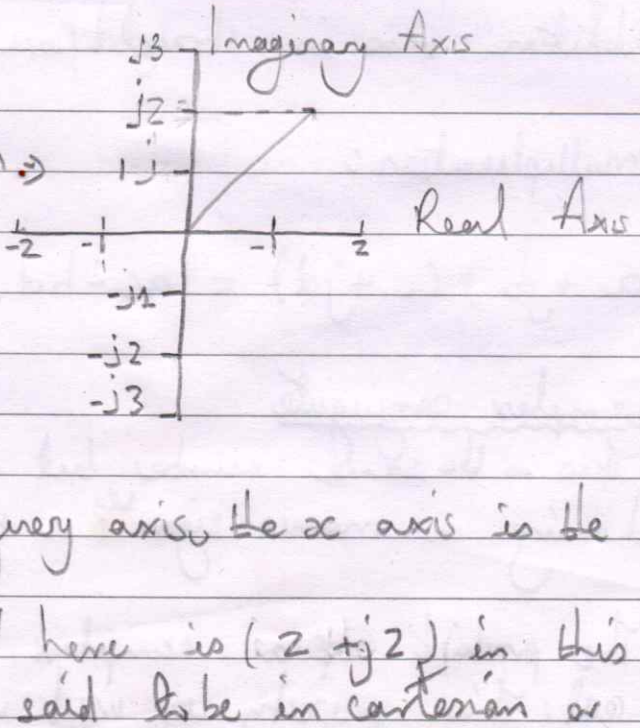
## Chapter 23 Revision of Complex Numbers

Introduction

Complex numbers can be used to represent anything that is periodic. Laplace transforms are used in control and fourier transforms are used to nalyse varying voltages and currents. ~~They~~ or Complex numbers are also extended into digital signal processing and digital image processing utilising the complex version of fourier analyses and wavelet analyses for the transmission, compression, restoration and other processing of digital audio signal, still images and video signals. These applications and many others are why the study of complex numbers is essential for digital analogue and so many engineering disciplines.

23.1

Argand diagram →




The y axis is the imaginary axis, the x axis is the real axis

The number represented here is $(2 + j2)$ in this written form it is said to be in cartesian or rectangular form.

Angle changes

## Angle Changes

An anticlockwise change of direction is an increase in angle. Multiplying by $j^1$ gives a 90 degree change of phase multiplying $j^2$ gives a 180° change of phase. Multiplying by $j^3$ gives a 270° change of phase and multiplying by $j^4$ gives a 360° change of phase anticlockwise.

∴ A change of phase (increase) of 90° results from a multiplication by ~~b~~ j a -90 clockwise change of phase results from a multiplication by -j.

~~by it~~ Proof can be seen by multiplying the number 5 + 0j by j and plotting the results on an argand diagram.

## Operations Involving Cartesian Complex Number 23.2 10-6-1

Addition : this is straightforward enough

Multiplication:

$$(a+jb)(c+jd) = (ac-bd) + j(ad+bc)$$

<u>Complex conjugate</u>
This is the same number but with the complex part having a minus sign ie $(a+jb)(a-jb) = a^2+b^2$

The product of a complex and its conjugate is always real; this property is used when dividing complex numbers.

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23.2 Operations involving Cartesian Complex Numbers 10-6-19

d Division

This is achieved by multiplying the numerator and denominator by the complex conjugate of the denominator. This makes the denominator real

$$\frac{1+2j}{3-4j} \times \frac{3+4j}{3+4j} = \frac{(1\times3-(2\times4)j)}{3^2+4^2} \quad \frac{-3-8j}{25}$$

0.12 + 0.2j

Rationalising is the elimination of the imaginary part of the denominator.

23.3 Complex Equations 10-6-19 ①

This section is about the equality of complex numbers, ie making one side equal to the other. If two complex numbers are equal then their real sides are equal so are their imaginary. This property is useful when deriving balance equations from ac bridges.

The only equation that is worth a mention is equations of this form.

$$R_1 R_3 = (R_2 + j\omega L_2)\left[\frac{1}{(1/R_4) + jWC}\right]$$

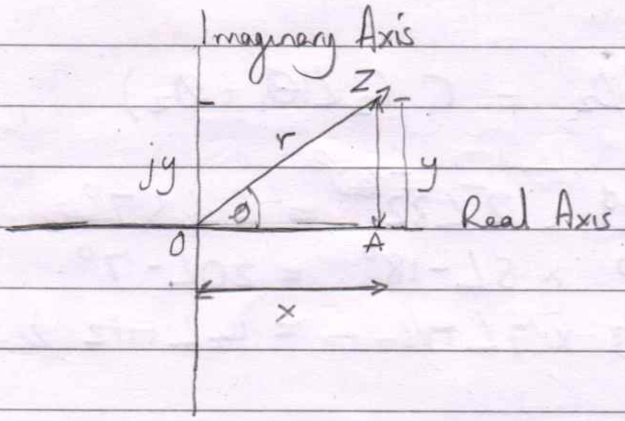
where

$R_1$, $R_3$, $R_4$ and $C_4$ are known find an expression for $R_2$ and $L_2$ in terms of the known components.

what i may start of by doing which would be wrong is finding the conjugate of the number on the right. Instead what i should do is multiply both sides by the denominator $(1/R_4 + jWC)$. Then the equation is manageable.

## 23.4 The Polar form of a Complex Number 10-6-19






$Z = x + jy = r\cos\theta + jr\sin\theta$

Z is usually abrevated to $Z = r\angle\theta$ which is called the polar form of a complex numbes.
r is called the modulus or magnitude of Z and is written mod Z or $|Z|$ r is determined by puthagorathes therem on the triangle OAZ

$$|Z| = r = \sqrt{(x^2 + y^2)}$$

$|Z|$ is the distance OZ.
θ is called the argument and is written as arg Z. θ is also deduced from the triangle OAZ $\arg Z = \theta = \tan^{-1}(y/x)$

$(3 + j4) = 5\angle 53.13^\circ$
$(-3 + j4) = 5\angle 180 - 53.13^\circ$

Polar 23.5

23.5 Multiplication and Division using Complex Numbers in Polar form

(a) $r_1 \angle \theta_1 \times r_2 \angle \theta_2 = r_1 r_2 \angle(\theta_1 + \theta_2)$

Thus $3 \angle 25^\circ \times 2 \angle 32^\circ = 6 \angle 57^\circ$

$4 \angle 11^\circ \times 5 \angle -18^\circ = 20 \angle -7^\circ$

$2 \angle \pi/3 \times 7 \angle \pi/6 = 14 \angle \pi/2$ )

(b) Division

$$\frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

The $P \to R$ and $R \to P$ button can be used to convert complex numbers from polar form to rectangular form and rectangular form to polar form

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23.6 DeMoivres theorem - powers and roots of Complex Numbers.

$|r\angle\theta|^n = r^n \angle n\theta$

This result is true for all negative positive or fractional values of $n$.

A square root of a complex number is.

$\sqrt{(r\angle\theta)} = r^{1/2}\angle\frac{1}{2}\theta$

it is however important to remember that a real number has roots which are 180° apart

In order to get the angle of the second root you add the original angle to 360 and divide by 2. Not forgetting to express the angle in a way that corresponds to an argand diagram ie angle no greater or less than ±180°.

$\sqrt{(12+j5)} = \sqrt{[13\angle 22.62°]}$ or $\sqrt{[13\angle 382.62°]}$

$= 13^{1/2}\angle\left(\frac{22.62}{2}\right)$ or $13^{1/2}\angle\left(\frac{382.62°}{2}\right)$

$= 13^{1/2}\angle 11.31$ or $13^{1/2}\angle \underline{191.32°}$

or $13^{1/2}\angle -168.69°$

24.1 Notes on Circuit Phase Angles

The phase angle of the circuit is the phase angle of the voltage - the phase angle of the current