Problem 15.

Complex waveform has an r.m.s value of 240 v and contains 30% in the 3rd Harmonic 10% in the fifth. Both of these harmonics initially being in phase with each other

(a) Determine r.m.s values of the fundamental and each harmonic

(b) Write down an expression to represent the complex voltage waveform if the frequency of the fundamental is 31.83 Hz.

(a) $V = \sqrt{(V_1^2 + V_3^2 + V_5^2)}$

$V_3 = 0.3V_1$, $V_5 = 0.10V_1$ and $V = 240V$

$240 = \sqrt{(V_1^2 + (0.3V_1)^2 + (0.10V_1)^2)}$

$240 = \sqrt{(1.10V_1^2)} = 1.049\ V_1$

$V_1 = \frac{240}{1.049} = \underline{228V}$

$V_3 = 0.3V_1 = 0.3(228.8) = 68.64V$

Rms value of the fifth harmonic

$V_5 = 0.10V_1 = (0.10)(228.8) = 22.88V$

(b) Maximum value of the fundamental $\sqrt{2} \times rms = V_{1m}$

$V_{1m} = \sqrt{2}\ V_1 = \sqrt{2}(228.8) = 323.6\ V$

Maximum value of the third Harmonic

$V_{3m} = \sqrt{2}\ V_3 = \sqrt{2}(68.64) = 97.07V$

Maximum value of fifth harmonic =

$V_{5m} = \sqrt{2}\ V_5 = \sqrt{2}(22.88) = 32.36V$

$v = 323.6 \sin 200t + 97.07 \sin 600t + 32.36 \sin 1000t)$ volts.

Exercise 127 Problem 3

3 $v(t) = 150 \sin(314t) + 40 \sin\left(942t - \frac{\pi}{2}\right) + 30 \sin(1570t + \pi)$

when integrating with respect to $d(wt)$ ie find

$\int_0^{\pi} v(t)\, d(wt)$ this above equations integral is written as

* (the above means replace (wt) with π and (w)) is the fundamental angular velocity

$$\frac{1}{\pi}\int_0^{\pi} 150 \sin(wt) + 40 \sin(3wt - \pi/2) + 30 \sin(5wt + \pi)\, d(wt)$$

where $w = 314$ rad/s

$$\frac{1}{\pi}\left[-150\cos(wt) - \frac{40\cos\left(3wt - \frac{\pi}{2}\right)}{3} - \frac{30\cos(5wt + \pi)}{5}\right]$$

$$\frac{1}{\pi}\left[(144) - -(144)\right] = \frac{288}{\pi} = 91.7 \text{ Volts}$$

instead of dt using the integral of $v(t)$ with respect to (wt). you can integrate it for the time period given by $\frac{1}{2f}$ = time in seconds and use that instead of π then multiply the integral by $2 \times f$.

3(c) form factor = $\frac{\text{r.m.s value}}{\text{mean value}}$

(25)

①

22/08/19

36.7 Power Associated With Complex Waves

The power of a complex wave is given by much the same way as the power in any circuit ie product of current and voltage or Current^2 x Resistance. The difference is we are dealing with complex waves. In order to get the power supplied we multiply the complex components of current by the complex component of voltage. As in the previous example where we found the root mean square of power the product of two complex components over an average cycle is 0, when they are of a different frequency. Therefore only products of voltages and currents of the same frequency matter.

for a current of value

$$i = I_{1m} \sin(\omega t - \phi_1) + I_{2m} \sin(2\omega t - \phi_2) + I_{3m} \sin(\omega t - \phi_3) + \ldots$$

$$v = (V_{1m} \sin \omega t + V_{2m} \sin 2\omega t + \ldots)(I_{1m} \sin(\omega t - \phi_1) + I_{2m} \sin(\omega t - \phi_2) + \ldots)$$

$$P = \int_0^{\pi} V_{1m} I_{1m} \sin(\omega t) \sin(\omega t - \phi_1) + V_{2m} \sin I_{2m} \sin(\omega t - \phi_2) + \ldots \, d(\omega t)$$

where P = average power

i = current output

v = voltage output

The average power $P_1 V$ over one cycle of the fundmental is given by:

$P_1 = V_1 I_1 \cos(\phi_1)$ where V_1 and I_1 are rms values of the first harmonic.

It follows that the average power over one cycle of the fundemental is given by

$P = V_1 I_1 \cos \phi_1 + V_2 I_2 \cos \phi_2 \ldots V_n I_n \cos \phi_n$ if there is a dc component to the voltage waveform add that ie add $V_0 I_0$ to P

(25)

If R is the equivalent series resistance of the circuit

$$P = I_0^2 R + I_1^2 R + I_2^2 R + I_3^2 R + \dots$$

Power factor

When dealing with harmonics the total power factor is defined as overall power =

$$= \frac{\text{total power supplied}}{\text{total r.m.s voltage} \times \text{total r.m.s current}} =$$

total power $= V_1 I_1 \cos \phi_1 + V_2 I_2 \cos \phi_2 \dots + V_n I_n \cos \phi_n$

where total power = sum of the average powers

total rms voltage $= \sqrt{\left(\frac{V_{1m}^2 + V_{2m}^2 + \dots V_{nm}^2}{2} \right)}$

total rms current = . . .

P

In the case where $i = A \sin(\omega t + \phi_1) + B \sin(B\omega t + \phi_2)$

$v = C \sin(\omega t + \phi_{v1}) + D \sin(B\omega t + \phi_{v2})$

then $P = \frac{A}{\sqrt{2}} \times \frac{C}{\sqrt{2}} \cos(\phi_{v1} - \phi_1) + \frac{B}{\sqrt{2}} \times \frac{D}{\sqrt{2}} \cos(\phi_{v2} - \phi_2)$

Problem 16

$i = (12 \sin(\omega t) + 5 \sin(3\omega t) + 2 \sin(5\omega t))$ amperes

find the average power in a 20Ω resistor if current i flows through it

$P = I_1^2 R + I_3^2 R + I_5^2 R$ $= I_{rms} \times R$

$= I \times R$ since

$\sqrt{\left(\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2\right)} = I$

$\left(\frac{12^2 + 5^2 + 2^2}{2}\right)^{1/2} = I = 9.3A$

$= 9.30 \times 20\Omega = 1730 W$

$1.730 kW$

Problem 17

A complex voltage

$v = 60 \sin \omega t + 15 \sin\left(3\omega t + \frac{\pi}{4}\right) + 10 \sin\left(5\omega t - \frac{\pi}{2}\right)$ volts

produces a complex current

$i = 2 \sin\left(\omega t - \frac{\pi}{6}\right) + 0.3 \sin\left(3\omega t - \frac{\pi}{12}\right) + 0.1 \sin\left(5\omega t - \frac{8\pi}{9}\right)$

a what is the active power (b) The power factor.

(a) (active power $= \Sigma$ average power

$P = V_1 I_1 \cos \phi_1 + V_3 I_3 \cos \phi_3 + V_5 I_5 \cos \phi_5$

$P = \frac{60}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \cos\left(+\frac{\pi}{6}\right) + \frac{15}{\sqrt{2}} \times \frac{0.3}{\sqrt{2}} \cos\left(\frac{\pi}{4} - \left(-\frac{\pi}{12}\right)\right) + \frac{10}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \cos\left(-\frac{\pi}{2} - -\frac{8\pi}{9}\right)$

(b) $P = 51.96 + 1.125 + 0.171 = 53.26 W$

Power factor $= \frac{\text{total average power}}{\text{total rms voltage} \times \text{total rms current}}$

total rms current $= I = \sqrt{\left(\frac{2^2 + 0.3^2 + 0.1^2}{2}\right)} = 1.43A$

total rms voltage $= V = \sqrt{\left(\frac{60^2 + 15^2 + 10^2}{2}\right)} = 44.30$

overall power factor $= \frac{53.26}{(44.30)(1.43)} = 0.841$

23/08/19 11:00AM

36.8 Harmonics in Single-phase circuits.

When a complex waveform containing harmonics is applied to a single phase circuit, which contains resistors capacitors and or inductors, ie a linear circuit; the result is also a ~~sine~~ harmonic wave.

For a voltage $V_{1m}\sin(\omega t) + V_{2m}\sin 2\omega t + V_{3m}\sin \omega t + \ldots$
The following scenarios can apply.

(a) If the circuit has an impedance of pure resistance, there is no phase change with respect to the current. The impedance of pure resistance is also independent of the frequency the general expression for current is given by.

$$i = \frac{v}{R} = \frac{V_{1m}\sin\omega t}{R} + \frac{V_{2m}\sin\omega t}{R} + \frac{V_{3m}\sin 3\omega t}{R} \ldots$$

The percentage of harmonic in the nth wave of the current wave is also the same as that of the voltage wave since $\dfrac{V_{2m}/R}{V_{1m}/R} \times 100\% = \dfrac{V_{2m}}{V_{1m}} \times 100\%$

(b) Pure inductance

$$i = \frac{v}{X_L} = \frac{V_{1m}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_{2m}}{2\omega L}\sin\left(2\omega t - \frac{\pi}{2}\right)$$
$$+ \frac{V_{3m}}{3\omega L}\sin\left(3\omega t - \frac{\pi}{2}\right) + \ldots$$

The impedance of a pure inductive reactance $X_L = 2\pi f L$ increases linearly with frequency. Also in every harmonic the current will lag the voltage by $-\frac{\pi}{2}$ or 90°.

Pure Capacitance

Capacitive reactance is given by $X_C = (1/(2\pi f C))$ $= 1/(WC)$. It varies with frequency in this way. Also the capacitive current leads the voltage by $\frac{\pi}{2}$ or 90° so capacitive current is given by

$$i = \frac{V}{X_C} = \frac{V_{1m}}{(1/WC)} \sin\left(wt + \frac{\pi}{2}\right) + \frac{V_{2m}}{1/(2WC)} \sin\left(2wt + \frac{\pi}{2}\right) + \frac{V_{3m}}{1/(3wC)} \sin\left(wt + \frac{\pi}{3}\right)$$

given the above we can see that

$$\frac{V_{1m}}{(1/WC)} = V_1 \times WC \quad \text{and} \quad \frac{V_{3m}}{(1/(3WC))} = V_{3m} \times 3WC$$

So for the nth harmonic the content of the sine wave will be n times larger than the fundemental

D.C. Components of V in Capacitance and Inductance Circuits

If a voltage contains a dc component in an inductance circuit, the direct current ~~voltage~~ drop will be zero across the pure inductance

If the DC component is contained in a pure capacitance circuit the ~~dit~~ dc current will not flow through the pure capacitor

23/08/19 36.8

31

Problem 19.

$v = (240 \sin 314t + 40 \sin 942t + 30 \sin 1570t)$ volts

$R = 12\Omega$ $L = 9.55\,mH$

(a) find an expression for i (b) the r.m.s voltage (c) The RMS current
(d) The power dissipated (e) the overall power factor.

Answer

(a) divide the first harmonic by $|Z_1|$ and subtract $\arg |Z_1|$ from $314t$ to get the first harmonic of current

Can't simply multiply Z_1 by 3 to get the third harmonic as the resistance remains the same and the reactance changes.

(d) Power dissipated is $(\Sigma(\text{Rms current}))^2 \times R$

(e) Overall power factor = $\dfrac{\text{total power}}{\Sigma\,\text{rms current} \times \Sigma\,\text{rms voltage}}$

1:48 36.9 24/08/19 Further Worked Problems on Harmonics in Single Phase Circuits.

Problem 21

$v = 25 + 100 \sin \omega t + 40 \sin(3\omega t + \frac{\pi}{6}) + 20 \sin(5\omega t + \frac{\pi}{12})$ volts

$\omega = 10^4$ rad/s is a series circuit consisting of a 5.0Ω resistor and 500μH inductance.

The current will consist of the dc voltage divided by resistance + the other components divided by their impedance

There is no dv components of voltage across the inductor but there is a dc current in the circuit. L

The solution is as before.

Problem 22

This voltage $v = 30 + 40 \sin 10^3 t + 25 \sin 2\times10^3 t + 15 \sin 4\times10^3 t$ volts produces this current

$i = 0.743 \sin(10^3 t + 1.190) + 0.781 \sin(2\times10^3 t + 0.896) +$
$+ 0.636 \sin(4\times10^3 t + 0.559)$ A

Determine (a) the average power supplied (b) The type of component present and (c) the value of the component

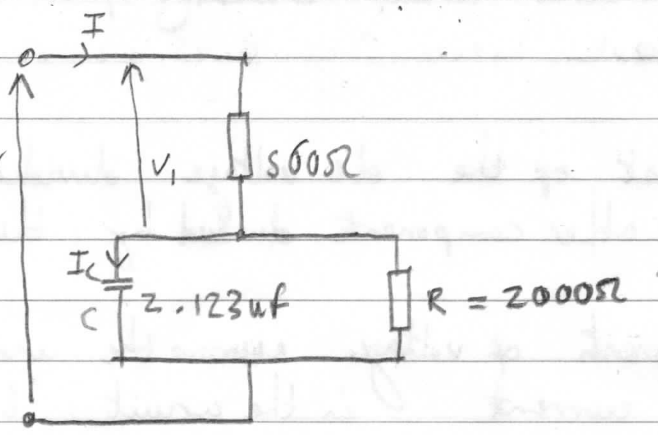
(a) The average power supplied can be worked out by the $\Sigma V_n I_n \cos \phi_n$ equation or the $(\Sigma I_n)^2 \times R$. The former works for parallel and series the latter only series

(b) The type of component is a capacitor as there is a V_0 but no I_0 a capacitor blocks out the dc current. Since power is delivered there must be a resistor present.

(c) The values of the component can be obtained from the impedance and the angular velocity ie $Z = R - jX_c$ $X_c = \frac{1}{\omega C}$

Problem 23

$v = 300 \sin 314t + 120 \sin (942t + 0.698)$ volts
In the circuit diagram shown below



(a) find an expression for the supply current (b) The % harmonic of the supply current (c) the total power dissipated (d) an expression for the p.d. shown in V_1 and (e) expression for the current through the capacitor.

(a) At the the fundemental the total impedance is given by Z_1

$$Z_1 = 560\Omega + \frac{-jX_{C_1} \times R}{-jX_{C_1} + R}$$

$$X_{C_1} = \left(\frac{1}{2\pi \times 50 \times 2.123uf}\right) = 1500\Omega$$

$$Z_1 = 560 + \frac{(2000 \times -j1500)}{(2000 - j1500)} = \frac{3 \times 10^6 \angle -90}{2500 \angle -36.87^\circ}$$

$$= \ldots \Rightarrow 1600 \angle -0.644\Omega$$

$$Z_3 = \left(\frac{2000 \times -j500}{2000 - j500}\right) = 825 \angle -0.607\Omega$$

$i_1 = \frac{300}{1600} \sin(314t + 0.644) =$

$i_3 = \frac{120}{825} \sin(942t + 0.698 + 0.607)$

$i = 0.1875 \sin(314t + 0.644) + 0.14545 \sin(942t + 1.305)$

(b) total active power dissipated / power flowing through the resistors

$= \left(\frac{I_{1m}^2 + I_{3m}^2}{2} \right) \times Z_1 \cos\phi_1$

$\left(\frac{0.1875^2 + 0.14545^2}{2} \right) \times 1600 \cos(-0.644)$

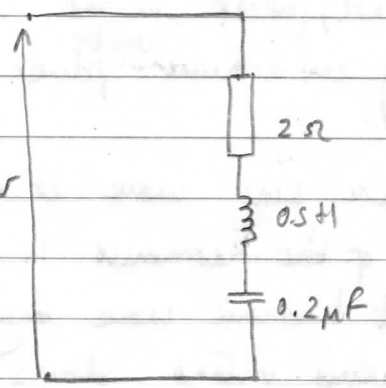
The above is wrong and will only work for series circuits or not at all. It is much better to work out the power by using $V_n I_n \cos\phi_n$ where ϕ_n is given by the sine angle - the cosine angle.

c find i_c by current division

25/08/19 Resonance Due to Harmonics 8:30 36:10

In industrial circuits at power frequencies the values of L and C make resonance at the fundamental rare, except in the capacitor start induction motor, where start-winding can achieve unity power factor during run up.
If the waveform is not a pure sine wave it is quite possible to achieve resonance at one of the harmonics. In which case the magnitude of the resonant harmonic wave may distort the current wave, causing dangerous voltage drops across L and or C.
When resonance occurs at one of the harmonic frequencies, the effect is called selective or harmonic resonance.
For resonance at the nth harmonic $n\omega L = \frac{1}{n\omega C}$

Problem 24



fundamental has a maximum value of 400V the 3rd harmonic has a maximum value of 10V. Determine (a) The fundamental frequency of the 3rd harmonic (b) The maximum value of the fundamental and third harmonic components of current.

(a) $3\omega L = \frac{1}{(3\omega C)}$ rearrange for f gives $\frac{1054}{2\pi} = 167.7\,Hz$

(b) find Z_1 and Z_3 ; divide maximum values of fundamental and 3rd harmonic by Z_1 then Z_3 for the latter

Problem 25

Maximum amplitude of voltage waveform is 800V fundamental is 50Hz and its nth harmonic has a maximum voltage amplitude of 1.5% of V_{1m}; Its in a series circuit with $R = 5\Omega$ $L = 0.369H$ and $C = 0.122\mu F$ resonance occurs at the nth harmonic.

(a) what is the value of n as in the nth harmonic. (b) I_{nm} c

(c) The the pd across the capacitor at the nth harmonic

(d) I_{1m}

(a) $n\omega L = \frac{1}{n\omega C}$ for resonance to occur at the nth harmonic

$n^2 = \frac{1}{\omega^2 LC} \Rightarrow n = \sqrt{\frac{1}{\omega^2 LC}} = n = \sqrt{\frac{1}{(2\pi \times 50)^2 \times 0.369 \times 0.122 \times 10^{-6}}}$

$n = 15.0023$

$n = 15.$

(b) at resonance the complex part is zero since $\left(n\omega L - \frac{1}{n\omega C}\right) = 0$

$V_{15m} = \frac{800 \times 1.5}{100} = 12V$

$R = 5\Omega \therefore I_{15m} = \frac{12}{5} = 2.4A$

(c) $I_{15m} \times X_{C15}$ = Pd across capacitor at nth harmonic.

$X_{C15} = 15 \times 2\pi f \times 0.122\mu F = 1.7394 k\Omega$

$2.4 \times X_{C15} = 4174.6 V = 4.1746 kV$

(d) $I_{1m} = \frac{V_{1m}}{Z_1} \Rightarrow Z_1 = 5 + j(X_{L1} - X_{C1})$

$X_{L1} = 2\pi \times 50 \times 0.369 = 115.925 \ \Omega$

$X_{C1} = \frac{1}{2\pi \times 50 \times 0.122\mu F} = 26.091 k\Omega$

$|Z_1| = |(5 + j(26.091k - 115.9))\Omega| = 2.5975 \times 10^4$

$I_{1m} = \frac{800V}{25975\Omega} = 0.0308 A$

Chapter 36 Complex Waveforms 12-8-19 Ex 123 - Ex 131 ①

Introduction 36.1

A waveform that is not sinusoidal is said to be a complex waveform. You can have periodic complex waveforms in which the for all value of t $f(t+T) = f(t)$ where T is the interval between two successive repetitions. T is called the period of the function.

A complex wave which is periodic can be resolved into the sum of a number of sinusoidal waveforms, and each of the sine waves can have a different frequency, amplitude and phase.

The initial sine wave component has a frequency f equal to the frequency of the complex wave and this frequency is called the fundamental frequency. The other sine wave components are known as harmonics, these having frequencys which are integer multiples of the fundamental frequency f. Hence the second harmonic is 2f, the third harmonic has a frequency of 3f and so on. If the fundamental frequency of a wave is 50Hz, then the third harmonic is 150Hz the fourth harmonic is 200Hz, and so on.

36.2 The General Equation of a Complex Waveform

The instantaneous value of a complex voltage wave v acting in a linear circuit may be represented by the general equation:

$$v = V_m \sin(\omega t + \psi_1) + V_{2m} \sin(2\omega t + \psi_2) + \ldots + V_{nm} \sin(n\omega t + \psi_n)$$

Here $V_{1m} \sin(\omega t + \psi_1)$ represent the fundamental component of which V_{1m} is the maximum or peak value, frequency,

eq ① $f = \omega / 2\pi$ and ψ_1 is the phase angle with respect to time, $t = 0$. Similarly $V_{2m} \sin(2\omega t + \psi_2)$ represent the second harmonic $V_{nm}(\sin n\omega t + \psi_n)$ represent the nth harmonic component, of which V_{nm} is the peak value, frequency $= n\omega/2\pi = nf$ and ψ_n is the phase angle.

In the same way the instantaneous value of current a complex current i may be represented by the general equation

eq ② $i = I_{1m} \sin(\omega t + \theta_1) + I_{2m} \sin(2\omega t + \theta_2) + \ldots + I_{nm} \sin(n\omega t + \theta_n)$ amperes

eq ① and eq ② refer to the voltage and current across a linear circuit, the phase angle between the fundamental voltage and current is $\varphi_1 = (\psi_1 - \theta_1)$
between the second harmonic and current is $\varphi_2 = \psi_2 - \theta_2$

Its often the case you have even harmonics in a complex wave or odd harmonic but not both.

RMS value $= \frac{1}{\sqrt{2}} \times$ Maximum Value

Mean Value $\frac{2}{\pi} \times$ Maximum Value

$y_2 = \sin(t + \psi)$ starts ψ earlier than $y_1 = \sin(t)$ so it is said to lead.

$y_2 = \sin(t - \psi)$ starts ψ radians later than $y_1 = \sin(t)$ so it is said to lag

Problem 1

A complex wave is given by: $v = 200\sin 100\pi t + 80\sin 300\pi t + 40\sin 500\pi t$ volts

Q (a) which harmonics are present (b) The RMS value of the fundemental
Q (c) The periodic time of the fundemental (e) the frequencys of the harmonics.

A (a) fundemental has an angular velocity of (100π) the next term has an angular velocity of 300π the last term has an angular velocity of 500π so the 3 and 5^{th} harmonics are present together with the fundemental.

A (b) The RMS value of the fundemental is $\frac{1}{\sqrt{2}} 200 = 141.4\,V$

A (c) frequency of fundemental $100\pi = \omega_1$

$= \omega = 2\pi f$

$\frac{\omega}{2\pi} = f = \frac{100\pi}{2\pi} = 50Hz$

(d) periodic time of fundemental $= \frac{1}{50Hz} = 0.02(S)$ or $20ms$

(e) frequency of third harmonic $3 \times$ frequency of first harmonic $= 150Hz$. frrequency of 5^{th} harmonic $= 50Hs \times 5 = 250Hz$

turn Over for Problem 2

Problem 2.

$i = 60 \sin 240\pi t + 24 \sin(480\pi t - \frac{\pi}{4}) + 15 \sin(720\pi t + \frac{\pi}{3})$ mA

(a) Determine the frequency of the fundamental (b) The percentage second harmonic (c) The percentage third harmonic (d) The RMS value of the second harmonic (e) The phase angles of the harmonic components and whether they are lagging or leading ~~(f) the mean value of the third harmonic~~
(f) The mean or average value of the third harmonic.

A (a) easy
(b) The percentage of the second harmonic means expressing the maximum value of the second harmonic as a percentage of the first harmonic:

$\frac{24}{60} \times 100 = 40\%$

(c) Percentage of the third harmonic is $\frac{15}{60} \times 100 = 25\%$
(d) Rms value of the second harmonic =

$\frac{1}{\sqrt{2}} \times 24 = 16.97$ mA

(e) The phase angles of the second harmonic is

$\frac{\pi}{4} = 45°$ leading at a guess

(e) The phase angle of the ~~seco~~ third harmonic is $\frac{\pi}{3}$ or $60°$. Lagging.

(f) The mean or average value is $0.637 \times$ max

$0.637 \times 15 = 9.56$ mA

14/8/19 (5) ⑦

General Conclusion on Example 1 to 6 (Odd Harmonics)

Whenever odd harmonics are added to a fundamental the positive and negative half cycle of the wave are identical in shape except for the sign change. This feature is true whether harmonics are added or subtracted from the fundamental.

General Rule for Even Harmonics.

Whenever even harmonics are added to a fundamental component:

(a) If the harmonics are initially in phase or if there is a phase shift of π rads, the negative half cycle, when reversed, is a mirror image of the positive half cycle about a vertical line drawn through time $t = T/2$

(b) If the harmonics are initially out of phase with each other by rather than by π radians (or 180°), the positive and negative half cycles are dissimilar

Examples used to get to this conclusion are:

① $i_b = 10 \sin \omega t + 4 \sin 2\omega t + 3 \sin 4\omega t$ amperes

② $i_c = 10 \sin \omega t + 4 \sin (2\omega t + \frac{\pi}{2})$ amperes

③ $i_d = 10 \sin \omega t + 4 \sin (2\omega t + \pi)$ amperes

Odd and Even Harmonics

(a) When a wave with odd and even harmonics is drawn and the harmonics are initially in phase the negative half cycle is a mirror image of the positive half cycle about the line $\frac{T}{2}$. For this to happen the positive and negative half cycle has to be multiplied by -1.

(b) If the harmonics are initially out of phase with each other, the positive and negative half cycles are not similar.

Examples used to illustrate this are:

$v_g = 50 \sin \omega t + 25 \sin 2\omega t + 15 \sin 3\omega t$ volts

$v_n = 50 \sin \omega t + 25 \sin (2\omega t - \pi) + 15 \sin (3\omega t + \frac{\pi}{2})$ volts

for a sort of half wave rectifier consider this circuit.

$i = 32 + 50 \sin \omega t + 20 \sin (2\omega t - \frac{\pi}{2})$ mA.

(7) (3)

Problem 3

$240 \sin(100\pi t) + 48 \sin(300\pi t + \frac{3\pi}{4})$

↓

240 rms $\Rightarrow 240 \times \sqrt{2}$ = peak $= 339.41$

$\Rightarrow 339.4 \sin(100\pi t) + 67.89 \sin(300\pi t + \frac{3\pi}{4}) =$

Practice Exercise 124

1

2 fundamental at 50 A rms, frequency = 100Hz, 24% third harmonic in phase at at $t=0$.

(a) Expression for this:

50 A rms $= 50 \times \sqrt{2} = V_m = 70.71$ V

$f = 100$ Hz $= \omega = 2\pi f = 200\pi$

fundamental $= 70.71 \sin(200\pi t)$

third harmonic $= 70.71 \times .24 = \underline{16.97}$ A $\quad \omega_3 = 600\pi$

$16.97 \sin(600\pi t)$

$(70.71 \sin(200\pi t) + 16.97 \sin(600\pi t))$ mA

2(a) fundamental ~~that~~ $v = 212.1$ V rms $\quad f = 50$Hz

2nd harmonic = 30% lagging by $(\pi/2)$ ~~to~~

4th harmonic = 10% leading by $(\pi/3)$

V_m fundamental $212.1 \times \sqrt{2} = 300$V $= 299.95$ V

$\omega_1 = 2\pi \times 50 = 100\pi$ Hz

fundamental $300 \sin(100\pi t)$ V

2nd harmonic $= 90 \sin(200\pi t - \frac{\pi}{2})$ V

4th $= 30 \sin(400\pi t + \frac{\pi}{3})$ V

$v = 300 \sin(100\pi t) + 90 \sin(200\pi t - \frac{\pi}{2}) + 30 \sin(400\pi t + \frac{\pi}{3})$

(4) fundamental : 16A frequency 1kHz
third and fifth harmonic $\frac{1}{5}$th and 1/10 the amplitude
all in phase at $t=0$

$16A \sin(2\times10^3\pi t) + 3.2 \sin(6\times10^3\pi t) + 1.6 \sin(10\times10^3\pi t)$

5 a Even or Odd and Even or Odd and Even out phase B 180°

(b) Odd
(c) Even out of phase or odd and even out of phase

(9) 5

36.4 Fourier Series of periodic and non periodic functions.

Introduction.

This is mainly used for analysing periodic functions into their constituent components. It is used for alternating current and voltages displacement, velocity and acceleration of slider crank mechanisms acoustic waves (as typical examples in engineering and science where periodic functions need analysis. The main topics are covered in this book, for more see Higher Engineering Maths, 7th Edition)

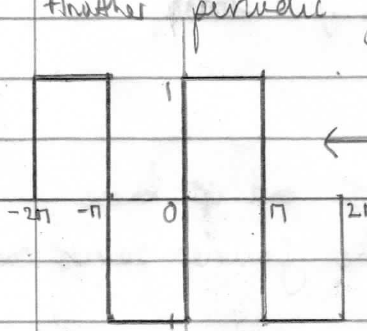
Periodic functions

A function is said to be periodic if $f(x+T) = f(x)$ for all values of x where T is the period.

for example $y = \sin(x)$ has a period of 2π doing the maths
$\omega = 2\pi f = 1 \Rightarrow \frac{1}{f} = \text{period } \frac{2\pi}{\omega} = \frac{2\pi}{1}$ since $\omega = 1$

This means that $\sin(x+2\pi) = \sin(x+4\pi) = \sin(x)$. If $y = \sin \omega t$ the period of the waveform is $\frac{2\pi}{\omega}$. And $\sin(2+2\pi) = \sin(2)$ $\sin(3+2\pi) = \sin(3)$ $\sin(1+2\pi) = \sin(1)$

Another periodic function is $f(x) = \begin{cases} -1 \text{ when } -\pi < x < 0 \\ 1 \text{ when } 0 < x < x \end{cases}$



$f(x) = -1$ when x is between $-\pi$ and 0 and =
$f(x) = 1$ when x is between 0 and π

This is an example of a function with finite discontinuities a function without these sudden jumps doesn't fit into this category and is called a continuous function
finite discontinuities means sudden jumps

The finite discontinuities are at $\ldots -2\pi, -\pi, 0, +\pi, +2\pi$ etc.
A great advantage of the fourier series is it can be applied to functions which are discontinuous and functions which are continuous.

The advantage of the fourier series is in expressing convergent functions which are defined in the interval $-x \leq x$ in terms of a convergent trigonometric series of the form:

$$f(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + \ldots \quad b_1 \sin(x) + b_2 \sin 2x + b_3 \sin 3x + \ldots \qquad (1)$$

when $a_0, a_1, a_2, \ldots b_1, b_2 \ldots$ are real constants,

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx)),$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \quad (n = 1, 2, 3, \ldots)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx \quad (n = 1, 2, 3, \ldots) \qquad (2)$$

a_0, a_n and b_n are called the fourier coefficients and if these can be determined, the series is called the fourier series corresponding to $f(x)$.
The term $(a_1 \cos x + b_1 \sin x)$ or $C_1 \sin(x + \alpha_1)$ is called the first harmonic or ~~con~~ fundamental the term $(a_2 \cos 2x + b_2 \sin 2x)$ or $C_2 \sin(2x + \alpha_2)$ is called the second harmonic. for an exact representation of complex waves an infinite number of terms is required in practice it is only necessary to take the first few.

Integrals

$\int \sin(x)\,dx = -\cos(x) + c$ $\quad\int \sin(3x)\,dx = -\dfrac{\cos(3x)}{3} + c$

$\int \cos(x)\,dx = \sin(x) + c$

The purpose of carrying out the operations in a fourier transform is to get the values $a_0, a_1 - a_n$ and b_1 to b_n by using equation 2 and put it in equation ①, to present the solution.

Continuing with our K example (page(s)) between 0 and π and $-\pi$ and 0 we have the following working to do.

$$\underline{\underline{a_0}} = \frac{1}{\underset{\text{period}}{2\pi}}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -k\,dx + \int_{0}^{\pi} k\,dx\right]$$

$$= \frac{1}{2\pi}\left\{\left[-kx\right]_{-\pi}^{0} + \left[kx\right]_{0}^{\pi}\right\} = 0$$

we get $\underline{a_0} = 0$ by inspection. a_0 is the mean value over a complete period of 2π. We can see this by sight

from equation ②

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -k\cos(nx)\,dx + \int_{0}^{\pi} k\cos nx\,dx\right\}$$

$$a_n = \frac{1}{\pi}\left\{\left[\frac{-k\sin nx}{n}\right]_{-\pi}^{0} + \left[\frac{k\sin nx}{n}\right]_{0}^{\pi}\right\} = 0$$

$$a_1 = \frac{k}{\pi}\left\{[-\sin 0 + \sin(-\pi)] + [\sin\pi - \sin 0]\right\} \quad [-\sin(-0) = \sin 0]$$

$$a_2 = \frac{k}{2\pi}\left\{[-\sin 0 + \sin(-2\pi)] + [\sin 2\pi - \sin 0]\right\} = 0$$

$$a_3 = \frac{k}{3\pi}\left\{[-\sin 0 + \sin(-3\pi)] + [\sin 3\pi - \sin 0]\right\} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -k\sin nx\,dx + \int_{0}^{\pi} k\sin nx\,dx\right\}$$

$$b_n = \frac{1}{\pi}\left\{\left[\frac{k\cos nx}{n}\right]_{-\pi}^{0} + \left[\frac{-k\cos nx}{n}\right]_{0}^{\pi}\right\}$$

$$b_1 = \frac{k}{1\times\pi}\left\{[\cos(0) - \cos(-\pi)] + -[+\cos(\pi) - \cos(0)]\right\}$$

$$b_1 = \frac{k}{\pi}(2 + - -2) = \frac{4k}{\pi}$$

$$b_2 = \frac{k}{\pi}\left[\frac{[\cos(0) - \cos(-2\pi)]}{2} + -\frac{[\cos(2\pi) - \cos(0)]}{2}\right]$$

$$b_2 = \frac{k}{2\pi}((1-1) + -((1) - (1))) = 0 \quad b_2 = 0$$

$$b_3 = \frac{k}{\pi}\left\{\frac{[\cos(0) - \cos(-3\pi)]}{3} + -\frac{[\cos(3\pi) - \cos(0)]}{3}\right\}$$

$$b_3 = \frac{k}{3\pi}(1 - -1) - (-1 - 1) = 2 + 2 \Rightarrow b_3 = \frac{4k}{3\pi}$$

$$b_4 = \frac{k}{\pi}\left\{\frac{[\cos(0) - \cos(-4\pi)]}{4} + -\frac{[\cos(4\pi) - \cos(0)]}{4}\right\}$$

$$b_4 = \frac{k}{4\pi} \times ((1-1) - (1-1)) = 0$$

$$b_5 = \frac{k}{\pi}\left\{\frac{[\cos(0) - \cos(-5\pi)]}{5} + -\frac{[\cos(5\pi) - \cos(0)]}{5}\right\}$$

$$b_5 = \frac{k}{5\pi}\left\{(1 - -1) - (-1 - 1)\right\} = \frac{4k}{5\pi} = b_5$$

putting it together with the equation on page 6

$a_0 = 0 \quad a_n = 0 \quad b_1 = \frac{4k}{\pi}, \; b_2 = 0, \; b_3 = \frac{4k}{3\pi} \quad b_4 = 0$

$b_5 = \frac{4k}{5\pi} \quad \therefore$

$$f(x) = \frac{4k}{\pi}\sin(x) + \frac{4k}{3\pi}\sin 3x + \frac{4k}{5\pi}\sin 5x \;\ldots\; \frac{4k}{\pi n}\sin(nx)$$

when n is odd

Expansion of Non Periodic Functions

If f(x) is not periodic then it cannot be expanded in a fourier series for all values of x, however it is possible to accurately represent the function within a period that is within 2π. This is little different from saying $f(x) = k$ within a range and $f(x) = -k$ within a range except that the function is something like $f(x) = x$. Of course this function can be repeated outside the range ~~as~~ of π and $-\pi$ as well as within it.

Problem 7

Determine the fourier series to represent the function $f(x) = 2x$ in the range $-\pi$ to π

for a fourier series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

from page 6 (4)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi} \times \left[\frac{2}{2} x^2\right]_{-\pi}^{\pi} =$$

$$a_0 = \frac{1}{2\pi}\left[(+\pi)^2 - (-\pi)^2\right] = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} 2x\cos(nx)\,dx$$

Integration by parts

$$\frac{2}{\pi}\left[\frac{x\sin(nx)}{n} - \int \frac{\sin(nx)}{n}\,dx\right]_{-\pi}^{\pi} = \frac{2}{\pi}\left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]$$

$$= \frac{2}{\pi}\left\{\left[0 + \frac{\cos n\pi}{n^2}\right] - \left[0 + \frac{\cos(n(-\pi))}{n^2}\right]\right\} = 0$$ for all n since $\cos(n\pi) = \cos(-n\pi)$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx \quad \text{where } n = 1, 2, 3, \ldots n$$

Integration by parts

$$b_n = \frac{2}{\pi}\left[\frac{-x\cos nx}{n} - \int\left(-\frac{\cos nx}{n}\right)dx\right]_{-\pi}^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-x\cos nx}{n} + \frac{\sin(nx)}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{2}{\pi}\left[\left(\frac{-\pi\cos n\pi}{n} + \frac{\sin n\pi}{n^2}\right) - \left(\frac{-(-\pi)\cos n(-\pi)}{n} + \frac{\sin n(-\pi)}{n^2}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n} - \frac{\pi\cos(-n\pi)}{n}\right]$$

$$= \frac{-4}{n}\cos n\pi \quad \text{since } \cos(-n\pi) = \cos(n\pi)$$

when n is odd $b_n = \frac{4}{n}$ thus $b_1 = 4$, $b_3 = \frac{4}{3}$ $b_5 = \frac{4}{5}$

when n is even $b_n = -\frac{4}{n}$ thus $b_2 = -\frac{4}{2}$ $b_4 = -\frac{4}{4}$

$b_6 = -\frac{4}{6}$, and so on

Thus, $f(x) = 2x = 4\sin x - \frac{4}{2}\sin 2x + \frac{4}{3}\sin 3x$
$- \frac{4}{4}\sin 4x + \frac{4}{5}\sin 5x - \frac{4}{6}\sin 6x$

ie $2x = 4\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + \frac{1}{5}\sin 5x \ldots\right.$

for values of $f(x)$ between $-\pi$ and π

For values of $f(x)$ outside this range $-\pi$ to π the sum of the series is not equal to $f(x)$.

17-08-19

Problem 8

Obtain a fourier series for $f(x) = \begin{cases} x, & \text{when } 0 < x < \pi \\ 0 & \text{when } \pi < x < 2\pi \end{cases}$

ie $f(x) = x$ between 0 and π and 0 between π and 2π

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

its better in this case to take the limits from 0 to 2π instead of from $-\pi$ to π. The fourier coefficient values are unaltered by this change in limits.

Hence

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi}\left[\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi} 0\,dx\right]$$

$$a_0 = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{1}{2\pi}\left(\frac{\pi^2}{2}\right) = \frac{\pi}{4}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\left[\int_0^{\pi} x\cos nx\,dx + \int 0\,dx\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{x\sin nx}{n} + \frac{\cos(nx)}{n^2}\right]_0^{\pi}$$ derived as in previous problem from integration by parts.

$$a_n = \frac{1}{\pi}\left\{\left[\frac{\pi\sin n\pi}{n} + \frac{\cos n\pi}{n^2}\right] - \left[0 + \frac{\cos 0}{n^2}\right]\right\}$$

$$a_n = \frac{1}{\pi n^2}(\cos(n\pi) - 1) \quad (\cos 0 = 1 \quad \sin \pi = 0)$$

when n is odd $=$ $a_n = \dfrac{-2}{\pi n^2}$ $\Rightarrow$ $a_1 = \frac{-2}{\pi}$ $a_3 = \frac{-2}{3^2\pi}$ $a_5 = \frac{-2}{5^2\pi}$

when n is even $=$ $a_n = 0$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\,dx \quad \text{for } n = 1, 2, 3, 4, 5, 6 \ldots$$

$$b_n = \left[\int_0^{\pi} x\sin(nx)\,dx - \int_{\pi}^{2\pi} 0\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{-x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right]_0^{\pi}$$

integration by parts

$$= \frac{1}{\pi}\left\{\left[\frac{-\pi\cos n\pi}{n} + \frac{\sin n\pi}{n^2}\right] - \left[0 + \frac{\sin 0}{n^2}\right]\right\}$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = \frac{-\cos(n\pi)}{n}$$

Since $\sin(0) = 0$ $\sin(\text{int} \times \pi = 0)$

here $b_1 = -\cos\pi = 1$ $b_2 = \frac{-1}{2}$ $b_3 = \frac{1}{3}$ and so on.

thus the fourier series is

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

ie $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\cos x - \frac{2}{3^2\pi}\cos 3x - \frac{2}{5^2\pi}\cos(5x)$

$\ldots \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \ldots$

the a_n terms can be abbreviated by putting

$$\sum_{n=1}^{\infty} a_n\cos(nx) = -\frac{2}{\pi}\left(\cos(x) + \frac{\cos(3x)}{3^2} + \frac{\cos 5x}{5^2} \ldots\right)$$

36.5 Even and Odd Fourier Series over any Range

Even functions

These have symmetry about the y axis so $f(-x) = f(x)$. $Y = \cos(x)$ is an example

Fourier Cosine Series

The fourier series of an even periodic function $f(x)$ having period 2π will only contain cosine terms, and may contain a constant term.

hence $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, d = 2 \times \frac{1}{2\pi}\int_0^{\pi} f(x)\, dx \Big\}\text{ due to symmetry.}$$

and $a_n =$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\, dx$$

Odd functions

A function $y = f(x)$ is said to be odd if $f(-x) = -f(x)$ for all values of x. Graphs of odd functions are always symmetrical about the origin. $y = \sin(x)$ is an odd function.
Many functions are neither even nor odd. The fourier series for an odd periodic function $f(x)$ has a period of 2π and contains sine terms only. Which means it also does not contain a constant term

hence $f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$

where $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$

$$b_n = \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin(nx)\,dx$$

Expansion of a Periodic function of Period L

It may be shown that if $f(x)$ is a periodic function with period L, then the fourier series is given by.

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n x}{L}\right) + b_n \sin\left(\frac{2\pi n x}{L}\right)\right]$$

when in the range $-\frac{L}{2}$ to $+\frac{L}{2}$. $a_0 = \frac{1}{L}\int_{-L/2}^{L/2} f(x)\,dx$

$$a_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\cos\left(\frac{2\pi \times n \times x}{L}\right)dx$$

$$b_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\sin\left(\frac{2\pi \times n \times x}{L}\right)dx$$

The limits of integration can be replaced by any interval of length L such as from $0 - L$

13/06/2020

fourier Series Extension of 36.4 from [illegible]

What are they

a fourier series is a way of representing a periodic and non periodic functions within a range of $\frac{2\pi}{L}$ where L is the period of the function.

In order to find a fourier series we get coefficients and multiply them by $a_1, \ldots, a_n \times \cos\left(\frac{2\pi n x}{L}\right)$

and $b_1, \ldots, b_n \times \sin\left(\frac{2\pi n x}{L}\right)$ we sum these products and add a_0.

The tricky part is getting the $a_0, \ldots, a_n$ coefficient and the $b_1, \ldots, b_n$ coefficient.

The Equation that gives the a coefficient is

$$a_0 = \frac{1}{2}\int_{-\pi}^{\pi} f(x)\,dx.$$

and $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \quad (n = 1, 2, 3, \ldots)$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx \quad (n = 1, 2, 3, \ldots)$$

when the wave is not square we often have to integrate by parts to find $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)$

and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = (n = 1, 2, 3, \ldots)$

21/08/19

36.6

36.6 R.m.s. value, mean value and form factor of a Complex Wave

If the instantaneous value of a complex current is given by

$i = I_{1m} \sin(\omega t + \theta_1) + I_{2m} \sin(\omega t + \theta_2) + \ldots I_{nm} \sin(n\omega t + \theta_n)$ amperes

The rms value of the first harmonic of current is given by

$I_1 = \frac{I_{1m}}{\sqrt{2}}$ where

using maths in section 36.6 it can be shown that that when you multiply harmonics together all but the square of the harmonic have a product equal to zero.

I is the symbol given to the mean value of i^2)

$I = \sqrt{(\text{mean value of } i^2)}$

$i^2 = [I_{1m} \sin(\omega t + \theta_1) + I_{2m} \sin(2\omega t + \theta_2) + \ldots I_{nm} \sin(3\omega t + \theta_3)]^2$

given we are finding the mean value of the square of terms the mean value of the square of the first harmonic will be given by $\frac{1}{2\pi}$ x integral of the first harmonic

Since the period we are measuring the harmonic over is 2π

$$\frac{1}{2\pi}\int_0^{2\pi} I_{1m}^2 \sin^2(\omega t + \theta_1)\, d(\omega t)$$

with some jiggery pockery as pages in section 36.6 this reduces to

$= \frac{I_{1m}^2}{2}$

hence it follows the mean value of $I_{nm}^2 \sin^2(n\omega t + \theta_n) = \frac{I_{nm}^2}{2}$

$i^2 = \frac{I_{1m}^2}{2} + \frac{I_{2m}^2}{2} + \ldots + \frac{I_{nm}^2}{2}$ hence the r.m.s value of current =

$$I = \sqrt{\left(\frac{I_{1m}^2}{2} + \frac{I_{2m}^2}{2} \ldots + \frac{I_{nm}^2}{2}\right)}$$

ie $$I = \sqrt{\left(\frac{I_{1m}^2 + I_{2m}^2 \ldots I_{nm}^2}{2}\right)}$$

or $I = \sqrt{(I_1^2 + I_2^2 + \ldots + I_n^2)}$

The Rms value for voltage is is also given by

$$V = \sqrt{\left(\frac{V_{1m}^2 + V_{2m}^2 + \ldots V_{nm}^2}{2}\right)}$$

$$\text{or } V = \sqrt{(V_1^2 + V_2^2 + \ldots V_n^2)}$$

8:38 from The Above Equations it can be seen that the r.m.s value of a complex wave is not affected by the relative phase angles of the harmonic components. for a dc components of a the complex wave I_0 or V_0 the mean rms and maximum values are all equal to each other. Thus the rms value I =

$$I = \sqrt{\left(I_0^2 + \frac{I_{1m}^2 + I_{2m}^2 + \ldots I_{nm}^2}{2}\right)}$$

$$I = \sqrt{(I_0^2 + I_1^2 + I_2^2 + \ldots I_n^2)}$$

Mean Value

This is measured over half a cycle for a complex quantity whose negative half cycle is similar to its positive half cycle.

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i \, d(\omega t)$$

21/08/19 36.6.

and for the voltage

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\,(d\,\omega t)$$

unlike rms values mean values are affected by relative phase angles of the harmonic components.

form factor.

The form factor of a complex wave whose negative half cycle is similar in shape to its positive half cycle is defined by

$$\text{form factor} = \frac{\text{r.m.s value of the waveform}}{\text{mean value}}$$

where the mean value is taken over half a cycle. changes in the phase displacement of the harmonics may appreciably alter the form factor of a complex wave.

Turn Over for. Relevant Problem.